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44 (Sem–4) M–II (HC–4026) N

2022

MATHEMATICS–II

Paper : BCA–HC–4026

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. (a) For the non-empty sets P, Q, R, prove that $P \cap (Q \Delta R) = (P \cap Q) \Delta (P \cap R)$. 3

(b) For the non-empty sets X & Y, prove that

$n(X \Delta Y) = n(X) + n(Y) - 2n(X \cap Y)$ 3

(c) In a class of 40 students, 22 have offered Mathematics, 9 have offered Mathematics but not Computer Science.

Now,

(i) how many of them have offered both Mathematics and Computer Science?

(ii) how many students how offered Computer Science?

(iii) how many students have offered Computer Science but not Mathematics? 2+1+2=5

(d) Define the term 'equivalence relation'.

Let $A = \{1, 2, 3, 4\}$

Consider the following relation

$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2,2), (2, 3)\}$

Examine R is transitive or not.

2+2=4

(e) Prove that

$$a + ar + ar^2 + ... + ar^n = \frac{ar^{n+1} - a}{r - 1},$$

if $r \neq 1$

[using mathematical induction] 3

(f) Let $h : \mathbb{Z} \to \mathbb{Z}$ be a function defined by $h(x) = 2x + 3$. And, $g : \mathbb{Z} \to \mathbb{Z}$ be a function defined by $g(x) = 3x + 2$. Find

(i) $g \circ h$

(ii) $h \circ g$ 4

2. (a) Define multigraph and pseudograph. Prove that the number of edges $E(G)$ is atmost $\frac{n(n-1)}{2}$ where G be simple graph with n vertices. 1+1+3=5

(b) Define Isomorphism of a graph with suitable example. 2+2=4

(c) Define rooted tree and decision tree. Prove that a tree with 'n' vertices there will be $(n-1)$ edges. 2+3=5

3. (a) In how many ways can the letters of the word EDINBURGH be arranged —

(i) with the vowels only in the odd places

(ii) beginning and ending with vowels.

(iii) beginning and ending with constants. 4

(b) How many diagonals are there in a polyon of 'n' sides? 3

(c) Find the value of ${}^nC_r + {}^nC_{r-1}$. 1

(d) There are 12 points on a plane, of which no three are collinear. If the points are joined, then

(i) how many straight lines can be obtained?

(ii) how many triangles can be formed?

2

(e) State Pigeonhole principle. 2

4. (a) Find the characteristic equation of

$$\begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$ 3

(b) If $A = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{pmatrix}$, find eigenvalues of

(i) A^{-1}

(ii) A^{200} 2+2=4

(c) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ 3

5. *(a)* Construct truth values for : 2+2=4

(i) $(p \wedge q) \wedge r$

(ii) $p \rightarrow (q \wedge r)$

2+4=6

(b)

Define the term 'Boolean Algebra'.

Express the following Boolean expressions E in terms of its minterm canonical form ***(any two)*** —

(i) $E = x(xy' + x'y + y'z)$

(ii) $E = (x + y'z)(y + z')$

(iii) $E = (x' + y)' + y'z$

6. *(a)* Define 'vector space' with suitable example. 2+2=4

(b) What do you mean by subspace and linear combination? 2+2=4

(c) Explain the basis and dimension of vector space. 4

Or

Show that the vectors $(0, 2, -4)$, $(1, -2, -1)$, $(1, -4, 3)$ are linearly dependent.